REMR Technical Note CS-ES-4.5

Uplift Pressures Resulting From Flow Along Tapered Rock Joints

Background

One of the key stages in a stability evaluation of navigation and flood-control structures is the calculation (or assignment) of uplift pressures along a critical rock joint or joints within the foundation of the hydraulic structure. Using accurate piezometric instrumentation data at a site along with knowledge of the site geology is the preferred method for establishing uplift pressures. However, when instrumentation data are not available or when the reservoir levels to be analyzed exceed those for which the piezometric measurements were made, other procedures must be used to establish the distribution of flow and the corresponding uplift pressures. Three procedures are widely used by engineers to establish the uplift pressures along an imaginary section or sections within the rock foundation. These three procedures are (1) a prescribed uplift distribution as given, for example, in an engineering manual specific to the particular hydraulic structure; (2) flow-net-computed uplift pressures; or (3) uplift pressures computed from flow within rock joints.

In 1992, investigators for the Electric Power Research Institute completed a study of 17 existing concrete gravity dams. The objective of this study was to identify key factors influencing uplift pressures. All dams were on instrumented rock foundations, and all had different foundation geology. An analysis of the uplift pressure measurements from each of these dams showed that foundation geology has a strong influence on uplift pressure distribution and that the geology controls the response of uplift pressure to changes in dam loading. The investigators discovered that an understanding of the flow within rock joints and the factors that affect the flow lead to a better understanding of the uplift measurements at the dam sites, especially those rock formations possessing "tight" rock joints.

Purpose

This technical note presents the results of a study involving one-dimensional, steady-state laminar flow through a single permeable joint within a rock foundation. Its purpose is twofold: to introduce the fundamentals of flow within rock joints and to show how the dimensions of the joint (referred to as joint aperture) influence the computed uplift pressures. Specifically, the results show the impact of a tapered aperture (i.e., constant change in taper with distance along a single rock joint) on the distributions of permeability and computed uplift pressures. The example considered is that of a horizontal rock joint located below the base of a concrete dam monolith for the cases of low, medium, and high reservoir elevations.

Modeling Joint Flow: The Cubic Law

Laminar flow within a rock joint can be characterized in a simplistic form as flow between a pair of smooth parallel plates separated by a constant distance. This distance is the joint opening or aperture, *e* (units of length). The flow rate per unit width is given by

$$Q = \frac{\gamma\, e^2}{12\, \mu} \cdot \left[- \frac{\partial h}{\partial l} \right] \cdot e \tag{1}$$

where γ is the unit weight of water (units of force per length cubed), *e* is the conducting aperture, and μ is the dynamic viscosity (e.g., lb-sec/ft^2 or slug/ft-sec in English units). The quantity of flow varies with the cube of the aperture *e*, hence the name "the cubic law." By analogy with Darcy's law, the equation for a single joint may be rewritten as

$$Q = K_{joint} \cdot [\, i \,] \cdot AREA_{flow} \tag{2}$$

where K_{joint} is the permeability, *i* is the hydraulic gradient, and $AREA_{flow}$ is the area of flow at any point along the single joint. Equation 2 can be used to compute the steady-state quantity of flow and distribution of uplift pressures given known values for γ and μ, the heads at each end of the joint, and the variation in aperture *e* with distance along the joint. Conventional one-dimensional steady-state seepage computer program packages that are commercially available can be used to perform the seepage analysis for any distribution of *e*.

In the special case of a tapered joint, it is possible to develop closed-form solutions for the quantity of flow within the joint and the distribution of uplift pressures along the length of the joint. These solutions are described in the following section.

Tapered Joint

A tapered joint, such as the one shown in Figure 1, is one that has linear variation in aperture with distance *x* along the joint (where *x* ranges in value from 0 to L). The equation for conducting aperture *e* is given as

$$e(x) = \left[\frac{e_{out} - e_{in}}{L} \right] * x + e_{in} \tag{3}$$

By Equation 1, the permeability at any point x varies in proportion to the square of the value of e

$$K_{joint}(x) = \frac{\gamma}{12\ \mu} \left[e(x) \right]^2 \tag{4}$$

The area of flow (per unit width) at any point along the joint is given as

$$Area_{flow} = e(x) \tag{5}$$

By introducing Equations 3, 4, and 5 into Equation 1 with $Q_{in} = Q(x) = Q_{out}$, and for the known head boundary conditions on either side of the joint as shown in Figure 1, the following relationships are obtained after some mathematical manipulations are performed:

$$Q = 2 \left[\frac{\gamma}{12\ \mu} \right] e_{in}^2 \left(h_{in} - h_{out} \right) \frac{1}{L} \left[\frac{e_{out}^2}{e_{out} + e_{in}} \right] \tag{6}$$

and

$$h(x) = h_{in} - \left[\left(h_{in} - h_{out} \right) \frac{1}{L} \left[\frac{e_{out}^2}{e_{out} + e_{in}} \right] \left[\frac{mx^2 + 2xe_{in}}{\left(mx + e_{in} \right)^2} \right] \right] \tag{7}$$

where

$$m = \frac{e_{out} - e_{in}}{L} \tag{8}$$

Equation 7 shows that the variation in head within a tapered joint is defined by five variables: the length of joint, the conducting apertures at the two ends of the joint, the reservoir head, and the tailwater head. Note that Equation 7 does not explicitly include the term K_{joint}.

Example Problem: Raising the Pool Behind a Gravity Dam Founded on a Single Rock Joint

The case of a single horizontal rock joint located below the base of a concrete monolith for the cases of low, medium, and high reservoir elevations is used to show the impact of joint aperture on uplift pressures. Figure 2 shows the hypothetical dam to be 300 ft high and 235 ft wide. It was assumed that jointing within the rock foundation was simplistic, i.e., a single rock joint parallel to and immediately below the dam-to-foundation interface. Changes in joint aperture during loading and/or unloading of the joint as a result of the construction of the dam and subsequent filling of the reservoir are not included in these calculations.

Three different tapers for the rock joint in Figure 2 were investigated using Equation 7: no taper, uniform aperture ($e_{in} = e_{out}$); taper downstream ($e_{in} > e_{out}$); and taper upstream ($e_{in} < e_{out}$). By assigning the datum to be the center line of the horizontal rock joint (Figure 2), the uplift pressure at any point is equal to the head at the point times the unit weight of water (with elevation head equal to zero and the velocity head being negligible).

The variation in head (and thus, uplift pressure) along the 235-ft-long rock joint is shown in Figure 3 for the pool elevations equal to 20, 150, and 300 ft for $e_{in} = e_{out} = 4.92 \times 10^{-4}$ ft (= 150 μm or 0.15 mm). This figure shows the uplift pressures to vary linearly along the joint for constant aperture.

Figure 4 shows the resulting variation in head with the joint tapered in the direction of flow (downstream) for the three pool elevations. In this example, e_{in} is set equal to $2e_{out}$, which results in the value of permeability at the toe (out) being one-fourth the magnitude of permeability at the heel (in). Comparison of the distribution of head or, equivalently, uplift pressure in Figure 4 with that shown in Figure 3 indicates that for a given pool elevation, a taper downstream results in larger uplift pressures compared to the case of uniform aperture.

Figure 5 shows the resulting variation in head with the joint tapered in the direction opposite to flow (upstream) for the three pool elevations. In this example, e_{in} is set equal to $e_{out}/2$, which results in the value of permeability at the toe being four times the magnitude of permeability at the heel. Comparison of the distribution of head or, equivalently, uplift pressure in Figure 5 with those shown in Figure 3 indicates that for a given pool elevation, a taper upstream results in smaller uplift pressures compared to the case of uniform aperture.

When the taper of the joint downstream is increased from a factor of 2 (Figure 4) to a factor of 10 (Figure 6), larger uplift pressures result. Conversely, when the taper of the joint upstream is decreased from a factor of 1/2 (Figure 5) to a factor of 1/10 (Figure 7), smaller uplift pressures result. Lastly, the results in Figure 8 show that in the case of a tapered joint, the ratio of e_{in} to e_{out} dictates the distribution of uplift pressures. The magnitudes of e_{in} and e_{out} impact the quantity of flow (see Equation 6).

Conclusions

The principal results of this study of laminar flow along a single horizontal tapered rock joint are as follows:

a. A uniform conducting aperture results in a linear variation in uplift pressures along the joint.

b. A taper downstream results in larger uplift pressures compared to the case of uniform aperture.

c. A taper upstream results in smaller uplift pressures compared to the case of uniform aperture.

d. The larger, or smaller, the ratio of e_{in} to e_{out} is from a value of 1.0, the greater the departure of the uplift distribution is from a linear relationship along the joint.

e. The magnitudes of e_{in} and e_{out} impact the quantity of flow.

Reference

Stone and Webster Engineering Corporation. (1992). "Uplift pressures, shear strengths, and tensile strengths for stability analysis of concrete gravity dams," report to Electric Power Research Institute, EPRI TR-100345s, Vol 1.

Acknowledgments

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Point of Contact

Name:	Dr. Robert M. Ebeling
Phone No.:	(601) 634-3458
Address:	Director U.S. Army Engineer Waterways Experiment Station ATTN: CEWES-IM-DI/Dr. Robert M. Ebeling 3909 Halls Ferry Road Vicksburg, MS 39180-6199

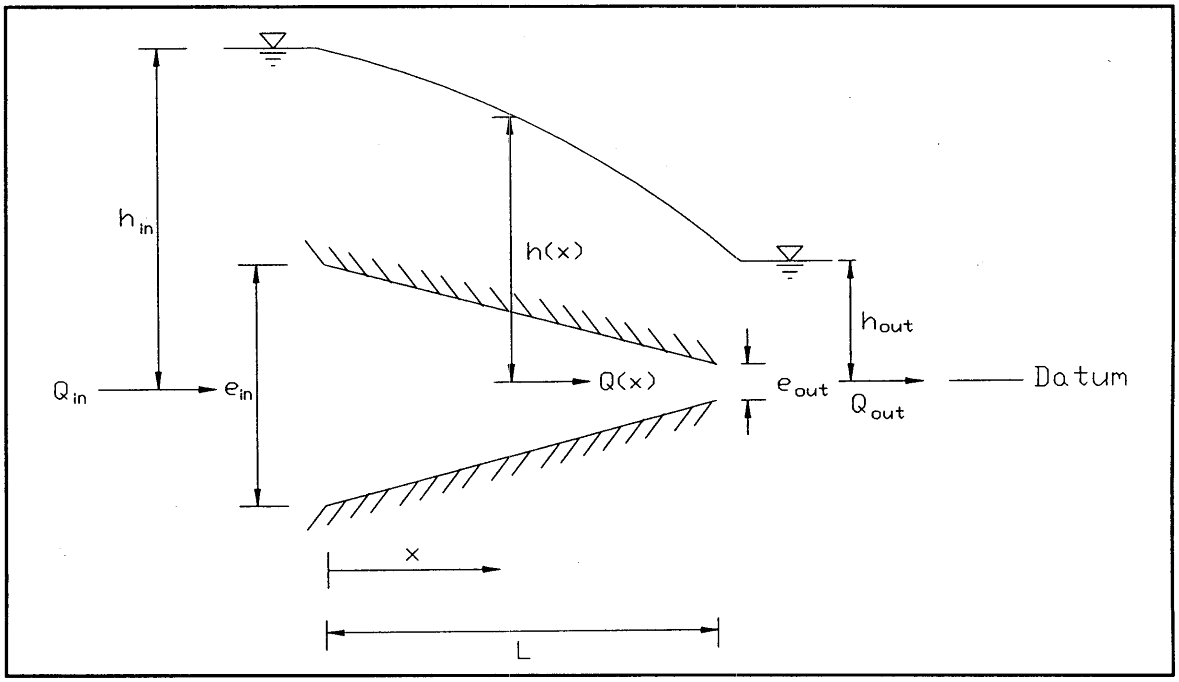


Figure 1. Variation of head along tapered joint as a function of position

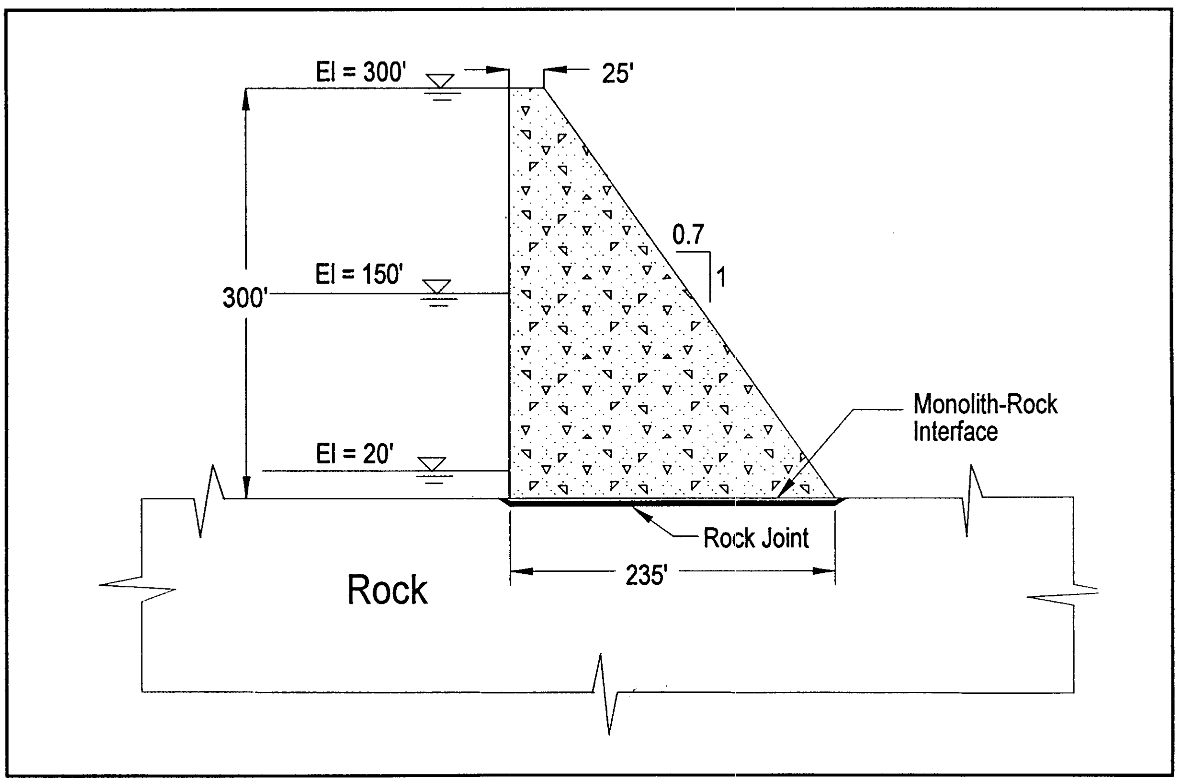


Figure 2. Geometry of dam used in study

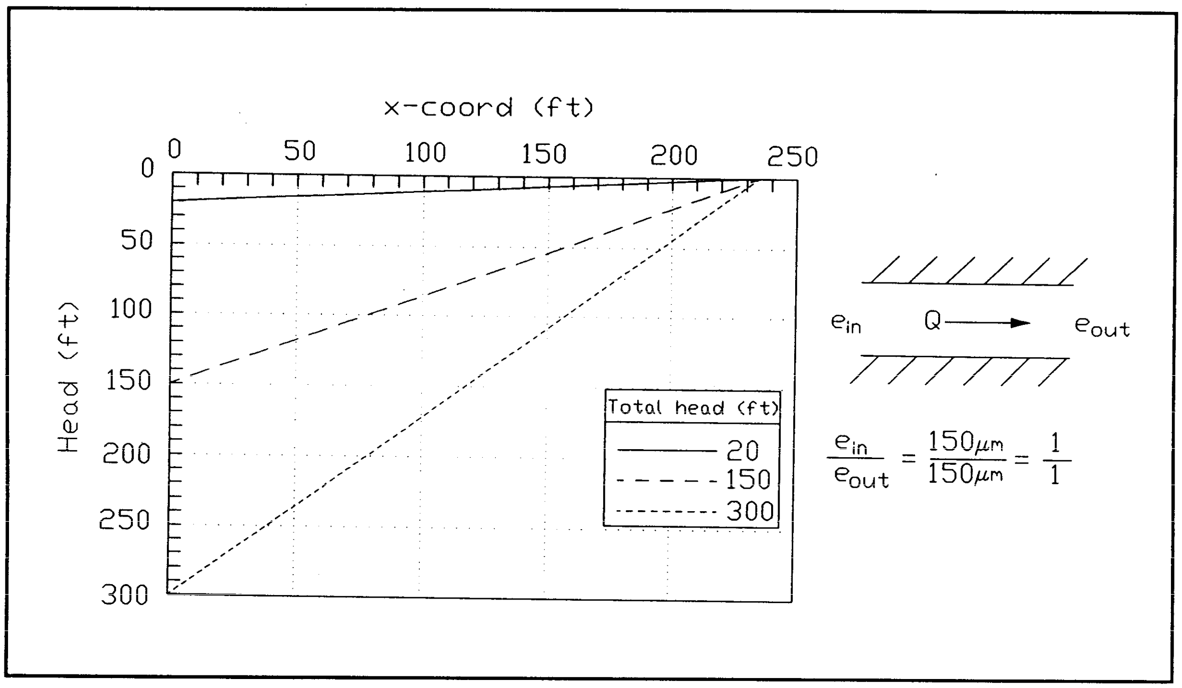
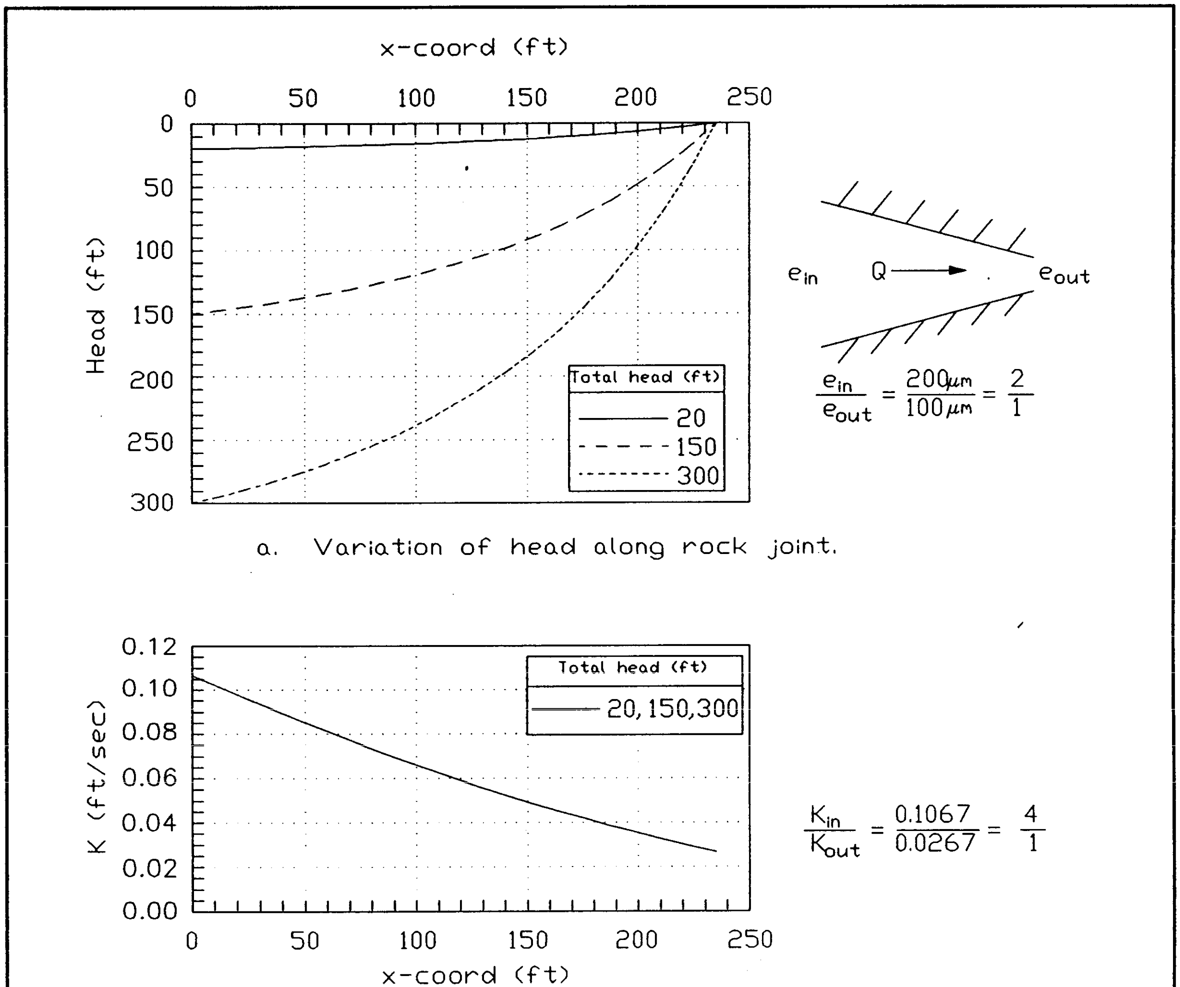


Figure 3. Variation in head along rock joint, $e_{in}/e_{out} = 1$



a. Variation of head along rock joint.

b. Variation of permeability along rock joint.

Figure 4. Head and permeability variation along rock joint, e_{in}/e_{out} = 2/1

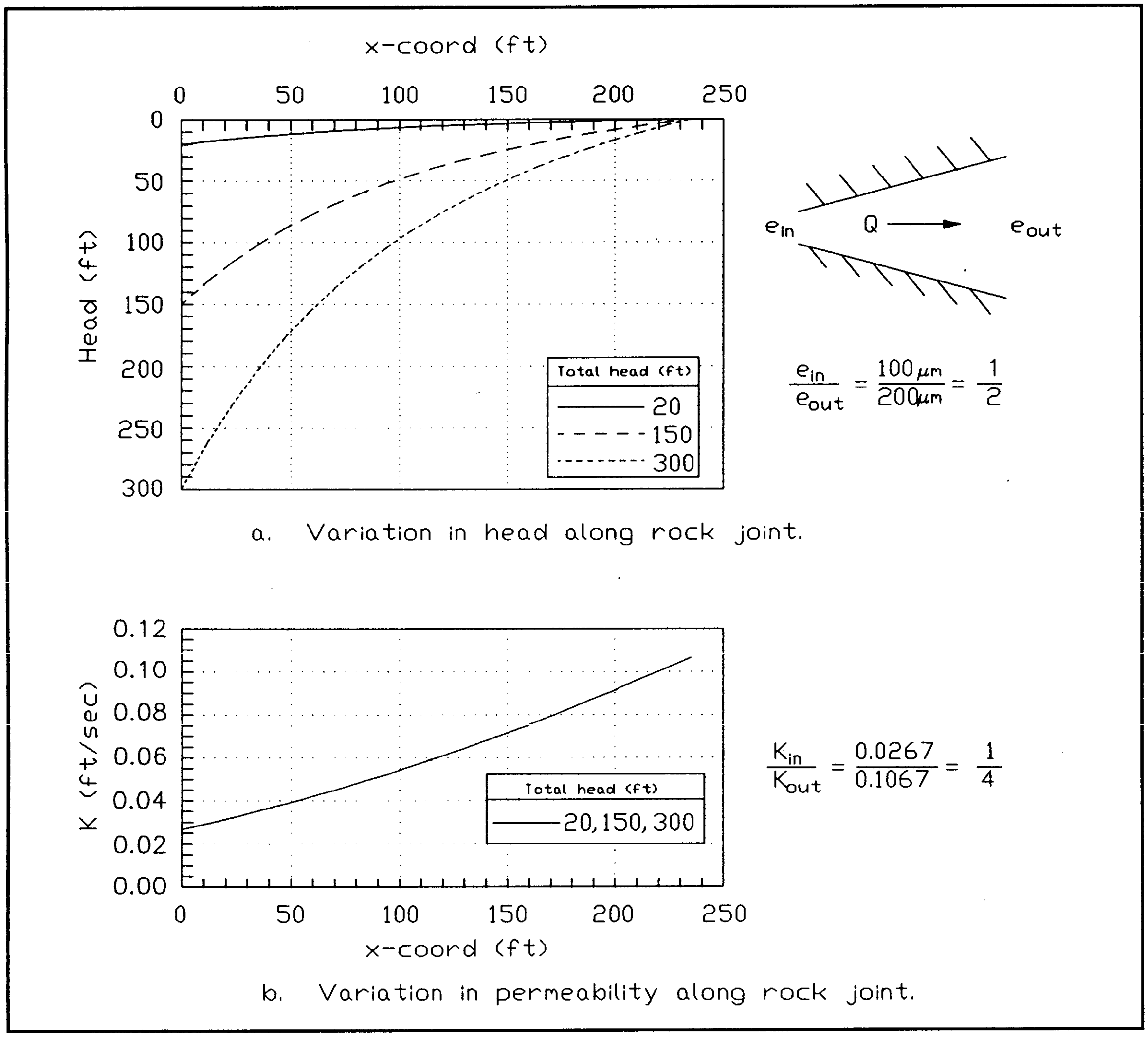
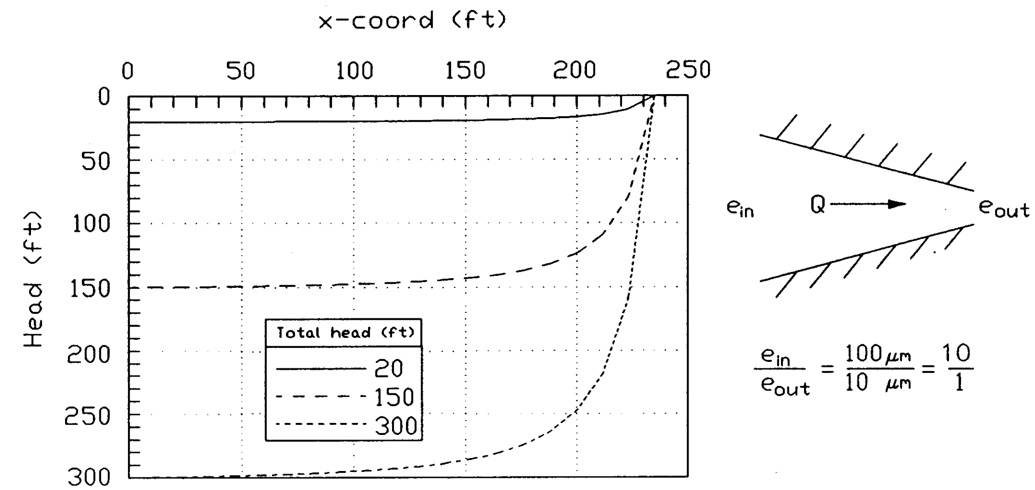
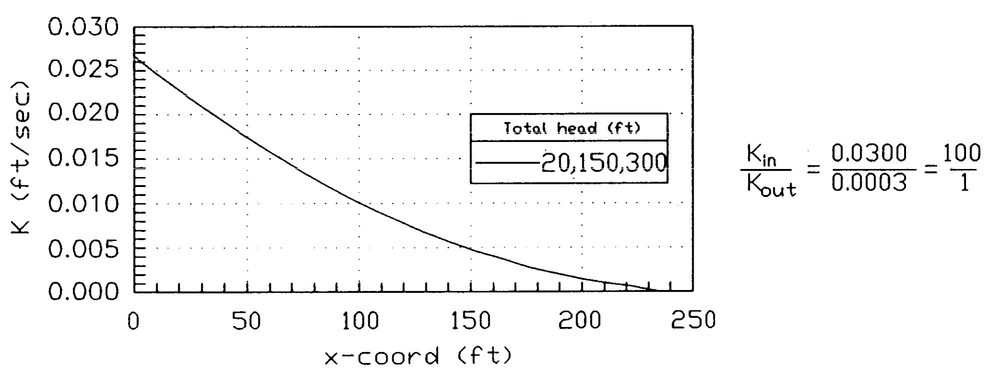


Figure 5. Head and permeability variation along rock joint, $e_{in}/e_{out} = 1/2$



a. Variation in head along rock joint.



b. Variation in permeability along rock joint.

Figure 6. Head and permeability variation along rock joint, e_{in}/e_{out} = 10/1

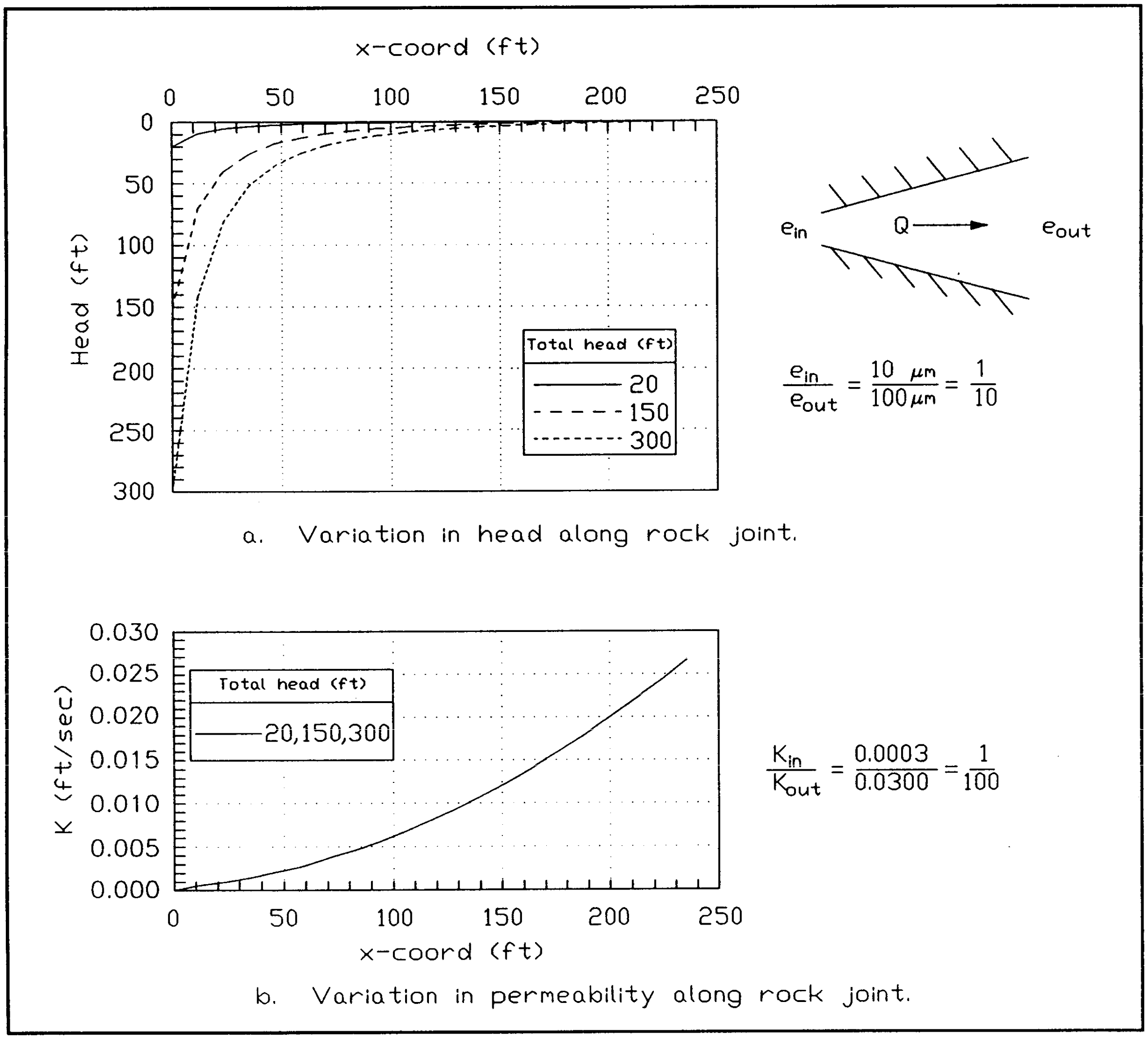


Figure 7. Head and permeability variation along rock joint, $e_{in}/e_{out} = 1/10$

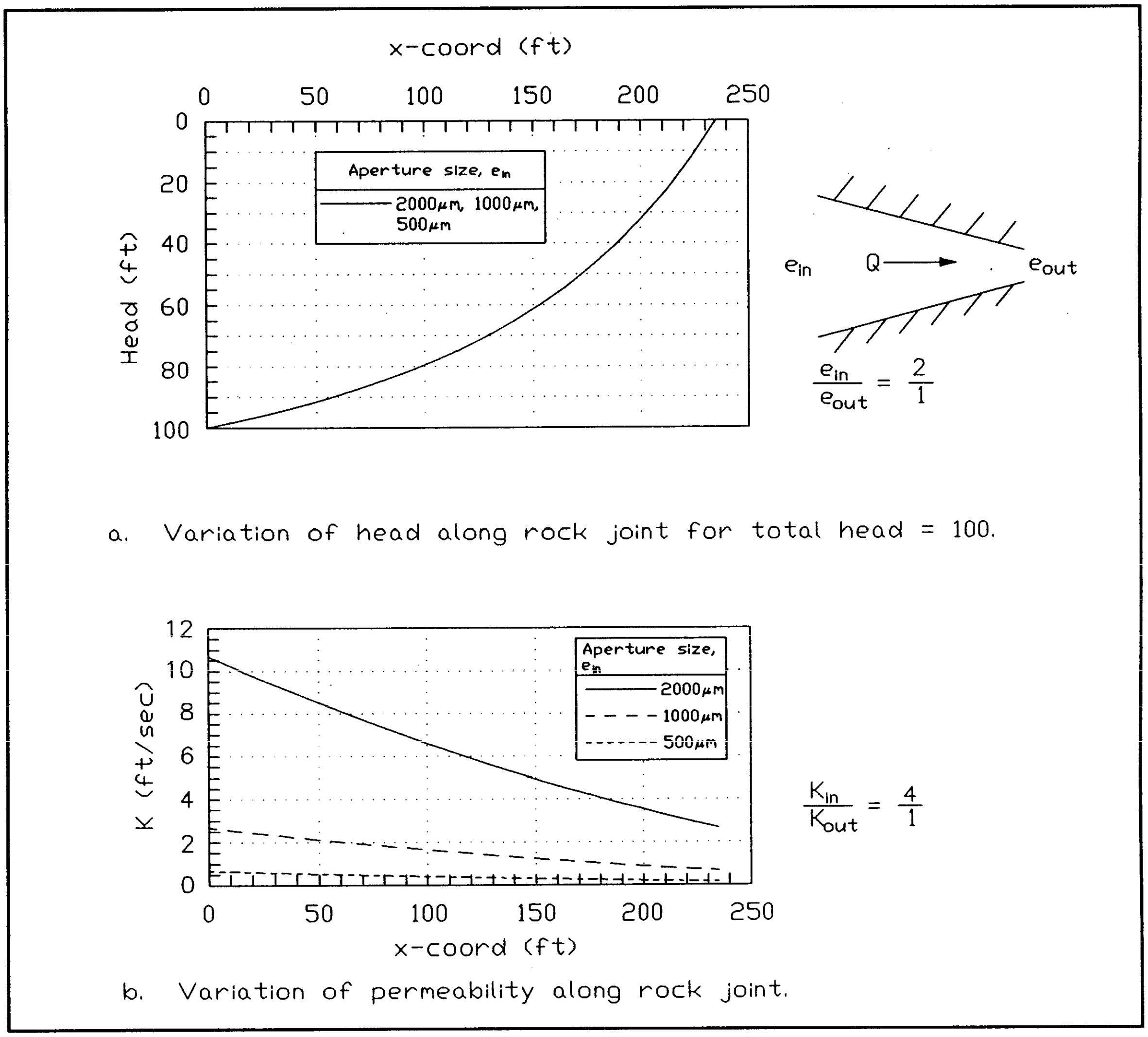


Figure 8. Head and permeability variation along rock joint, e_{in}/e_{out} = 2/1, with varying e_{in}